Scalar Integration in Space Multivariable Calculus X. Du

- **Scalar Line Integrals**
  - $\int_C f ds$
  - The scalar line integral of a function *f* along a piecewise smooth curve *C* parameterized $\vec{r}(t)$ is $\int_C f ds = \int_{t_1}^{t_2} f(\vec{r}(t))|\vec{r}(t)|dt$.
    - This is just a simple substitution. Recall that $\frac{ds}{dt} = |\vec{r}\,'(t)|$. Therefore, we can substitute $ds = |\vec{r}\,'(t)|dt$
  - $\int_C ds$ is the length of the curve *C* in space.
  - If $f(x, y)$ is a function in space, then $\int_C f ds$ is the area under the curve above the xy-plane.
  - Does not depend on orientation!
  - Applications: mass, total charge of a wire, area of a curvy fence.
- **Scalar Surface Integrals in Three-Space**
  - $\iint_S f dS$
  - The scalar surface integral of a function *f* through a smooth surface *S* parameterized $\vec{r}(s,t)$ is $\iint_S f dS = \iint_D f(\vec{r}(s,t))|\vec{r}_s \times \vec{r}_t|dA$, where *D* is the domain of *S* in the st-plane.
    - This is just a simple substitution. $dS = |\vec{r}_s ds \times \vec{r}_t dt|$ since *dS* can be approximated as an area of a parallelogram as linear approximation is a good approximation since *dS* is infinitely small. Thus, $dS = |\vec{r}_s ds \times \vec{r}_t dt| = |\vec{r}_s \times \vec{r}_t| ds dt = |\vec{r}_s \times \vec{r}_t| dA$, where *dA* lives in the st-plane. Therefore, we can substitute $dS = |\vec{r}_s \times \vec{r}_t| dA$.
  - If you can evaluate the surface integral in the st-plane, no Jacobian is necessary! The Jacobian is only necessary when utilizing a change of variables.
  - $\iint_S dS$ is the surface area of *S* in space.
  - Does not depend on orientation!
  - Applications: mass, total charge of a lamina.